Nonlinear Statistical models for COVID-19 fourth wave cases in Egypt

النماذج الإحصائية غير الخطية لبيانات الموجة الرابعة من COVID-19 في مصر

Dr. Maha Farouk

Department of Statistics, Mathematics and Insurance- Faculty of Commerce – Tanta University

Maha.ibrahim@commerce.tanta.edu.eg

مجلة الدراسات التجارية المعاصرة

كلية التجارة – جامعة كفر الشيخ

المجلد التاسع . العدد الخامس عشر- الجزء الأول

يناير 2023

رابط المجلة : **https://csj.journals.ekb.eg**

Abstract

This study aimed to apply the nonlinear models (Gompertz, Richards', and Weibull) that enables us to study and forecast the daily number of COVID-19 cases in Egypt and determine which of them is the suitable to describe the data of COVID-19 fourth wave cases during the period from 28th of July, 2021 to 5 th January, 2022.The models' parameters were estimated, and the Comparison of these models' fits was made using some statistics (F-test,R^2, AIC, BIC, AICc, and MAPE). Applied on "nonlinear regression" tool available in SPSS-26, and Microsoft excel 2016. According to the highest F and R^2, and lowest values for RMSE, Bias, MAE, AIC, BIC, and AICc, the results indicate that the Weibull model is the best adequate model for studying the daily number of COVID-19 cases in Egypt. The proposed Weibull model is statistically significant for describing the study data.

Keywords: COVID- 19; non- linear regression; growth models; Weibull model; Gompertz model; Richards' model; Epidemic models; Initial Parameters

المستخلص :

هدفت هذه الدراسة إلى تطبيق النماذج غير الخطية (Gompertz و Richards و Weibull) التي تمكننا من دراسة وتوقع العدد اليومي لحالات COVID-19 في مصر وتحديد أي منها مناسب لوصف بيانات حالات الموجة الرابعة COVID-19 خلال الفترة من 28 يوليو 2021 إلى 5 يناير 2022.

تم تقدير معلمات النماذج ، كماتم إجراء مقارنة مناسبة هذه النماذج باستخدام بعض الإحصائيات) اختبار F، R2 ، AIC، BIC وAICc و(MAPE باستخدام أداة "الانحدار غير الخطي" المتوفرة في-SPSS 26وMicrosoft Excel 2016.

تشير النتائج إلى أن نموذج Weibull ذو دلالة إحصائية لوصف بيانات الدراسة وبذلك هو أفضل نموذج مناسب لدراسة العدد اليومي لحالات COVID-19 في مصرحيث اظهرت تنائج نموذج Weibull المقترح أعلى قيم F و R2 وأدنى قيم لـ RMSE و Bias و MAE و AIC و BIC وAICc

الكلمات المفتاحية : COVID- 19؛ الانحدار غير الخطي ؛ نماذج النمو , نموذج ويبل , نموذج جومبيرتز , نموذج ريتشاردز ؛ النماذج الوبائية , المعلمات الأولية

1. Introduction

(COVID-19) is one of the biggest public health crises that has ever faced the world. So, to fight the disease every country must have at least some effective models to describe the different stages of the epidemic's evolution as a guide for the authorities to take appropriate measures. (Fahmy, et al, (2020)) It infects a lot of individuals in Egypt. Studying of the spread of disease, using of Statistical models in public health decision making has become increasingly important to control and prevent further outbreaks, and limiting their devastating effects on a population. This study aims to describes the daily numbers of the fourth wave of COVID-19 confirmed cases in Egypt as a growth model using three proposed Nonlinear Statistical modeling (Weibull model; Gompertz model; Richards' model), and determine which of them will be statistically significant for describing the COVID-19 confirmed cases in Egypt data.

In Egypt, the first COVID-19 case was declared on 14 th February 2020. At18th February 2022, we get the peak of the fourth wave of COVID-19. as daily reports were published by the Health and Population Ministry, the daily infections exceed 2035 with cumulative infections 465423 and about 52 daily deaths with cumulative deaths 23632 had been reported (https://www.care.gov.eg/EgyptCare/index.aspx)

Generally, to study the infectious of diseases there are three kinds of methods. (AL-Ani (2021))

(1) Using Dynamic model to establish of infectious diseases

(2) Building of statistical modeling of infectious diseases data based on statistical methods.

(3) Find the epidemic law of infectious diseases by using data mining methodology to obtain the information in the data.

Governments need some statistical models to be able to predict the infected cases in the coming period. Nonlinear Statistical models is very important Statistical tools. So, (Shafii, et al,1991) had a nonlinear estimation of growth curve models for germination data analysis, Rodr´ıguez,(2010) had a Parametric Survival Models using Survival Distributions , Szabelska (2010) made some comparison of growth models in package R, Dagogo (2020) also made comparative Analysis of Richards, Gompertz and Weibull Models on an experiment conducted at the university of Port-Department Harcourt's of electrical/electronic engineering to determine the quantity of transferred voltage as a function of time, (Fahmy, et al (2020)) got a modified variant of SEIR(Susceptible, Infectious, Exposed, and Recovered) is implemented to predict the behavior of COVID-19 in epidemic analysis in Egypt, Qatar and Saudi Arabia using the Generalized SEIR Model , (Amar ,et al,(2021)) Prediction of the final size for COVID-19 epidemic using machine learning: a case study of the second wave COVID-19 in Egypt, (Deif , et al (2021) modeling the COVID-19 spread, a case study of Egypt, (Mansour , et al (2021))modeling the COVID-19 Pandemic dynamics in Egypt and Saudi Arabia, Al-Ani(2021) had a statistical modeling of the novel COVID-19 epidemic in Iraq, and (Radwan (2021)) introduced a time series analysis using autoregressive integrated moving average (ARIMA)to get a statistical study of COVID-19 pandemic in Egypt .

2. Nonlinear growth models

The increases of height, length or other specifications may be required is described by growth curve models depending on the type of phenomenon studied with relation to time. Nonlinear regression models are important tools because many processes are represented by nonlinear better than linear models. A nonlinear growth model is one in which at least one of the parameters appears nonlinearly. the parameter is nonlinear, if the second derivative of the function with respect to a parameter is not equal to zero. Nonlinear statistical models have been used to describe growth behavior that varies in time. Depending on the type of that occurs, the type of model needed in a specific area or specific situation. Inflection point is the point at which the rate of growth gets maximum value. The general form of the growth models or nonlinear models is (Leite ,2018)

$$y_t = f(t;\theta\ ) + \varepsilon_t, \qquad (t = 1, 2, 3,\dots, n)$$

where:

y_t is the dependent or response variable,

t is the time (independent variable),

$\boldsymbol{\theta}$ is the vector of unknown p-parameters such that

$\boldsymbol{\theta} = (\theta 1, \theta 2, \ldots, \theta p)'$

ε_t is a random error term and $\varepsilon_t \sim$ NID $(0, \sigma^2 \varepsilon)$.

It is not possible to have an exactly solution of nonlinear equations. By employing iterative procedures such linearization (or Taylor series) method, Steepest Descent method, and Levenberg-Marquardt's method to obtain approximate analytic solutions. The nature relation between y_t and t is not linear, and the goal is to estimate θ_j ′ s by nonlinear ordinary least squares (OLS) which minimizing the sum of squares residual (SSR) function. In fitting nonlinear models, the following steps are considered: (Archontoulis, (2015))

1. Choose candidate models, and set starting values.
2. Fit models, and check convergence and parameter estimates,
3. Find the "best" model among competing models
4. Check model assumptions (residual analysis)
5. Calculate statistical descriptors and confidence intervals

When the data functions are Sigmoid functions, many models often used especially: Gompertz model, Richards' model and Weibull model.

Gompertz model:

Gompertz model has sigmoid type of behavior and it is found quite use full in the biological work. It is asymmetrical about the inflection point with a fixed skewness of approximately 1.3, and not symmetric about its point of inflexion. The Gompertz function is a nonlinear growth model describes growth that of a given time period, starts slow and ends slow. It is a special case of the generalized logistic model (GLM). The dependent variable y_t is a cumulative percentage at time t, Gompertz function with all the parameters of these model are of positive values is formed as (Szabelska, et al, (2010))

$$y_i = \beta_0 (e^{-\beta_1 e^{-\beta_2 x_i^{\beta_3}}}) + \varepsilon_t,$$

Were

β_0 maximum growth response or scale parameter when time approaches ∞

β_1 the shape parameter related to initial time

β_2 growth range (or intrinsic growth range)

β_3 growth rate

x_i represents time

y_i is the i[th] observation at specific time

Table (1) show the special cases of Gompertz model

Table (1): the special cases of Gompertz model

case	the initial value of y_i
x_i=0, or ($\beta_2 = 0$),or($\beta_3 = 0$)	$y_i = \beta_0(e^{-\beta_1})$
($\beta_3 = 1$)	$y_i = \beta_0(e^{-\beta_1 e^{-\beta_2 x_i}})$
($\beta_1 = 0$)	$y_i = \beta_0$
($\beta_1 = 1$)	$y_i = \beta_0(e^{-e^{-\beta_2 x_i{}^{\beta_3}}})$
($\beta_2 = 1$)	$y_i = \beta_0(e^{-\beta_1 e^{-x_i{}^{\beta_3}}})$

The left-hand valued (lower value) is approached less gradually by the curve than the right-hand value (future value). The inflection point of Gompertz equation has controlled by its asymptotic value and is at about (e^{-1} = 0.3679). In fact, the Gompertz is a log-Weibull distribution. However, unlike logistics model, this is not symmetric about its point of inflexion. This distribution provides a remarkably close fit to adult mortality in contemporary developed countries (Rodr´ıguez, (2010))

Richards' model

The Richards' model is a sigmoid function based on a four-parameter, with β_1, the shape parameter, measuring the various patterns and indicating alternative functional forms (i.e., Gompertz, logistic, monomolecular) that may be generated by this model. The Richards' model is formed as (Archontoulis, et al, (2015))

$$y_i = \beta_0(1 - \beta_1 e^{-\beta_2 x_i})^{\beta_3} + \varepsilon_t,$$

β_3 is the inflection parameter, which determines the function shape. Table (2) shows the special cases of Richards' model

Table (2) :the special cases of Richards' model

case	the initial value of y_i
x_i=0, or ($\beta_2 = 0$)	$y_i = \beta_0(1 - \beta_1)^{\beta_3}$
($\beta_3 = 0$), or ($\beta_1 = 0$)	$y_i = \beta_0$
($\beta_3 = 1$)	$\beta_0(1 - \beta_1 e^{-\beta_2 x_i})$
($\beta_1 = 1$)	$y_i = \beta_0(1 - e^{-\beta_2 x_i})^{\beta_3}$
($\beta_2 = 1$)	$y_i = \beta_0(1 - \beta_1 e^{-x_i})^{\beta_3}$

The Richards' model is more flexibility in dealing with asymmetric growth, the inflection point can be at any x value.

Weibull model:

The Weibull growth model is useful for modelling growth instability in particular. So, with great potential for application to biological data, the Weibull model is a flexible and simple function, and its first introduced was as a statistical distribution. Weibull model is used to describe disease, survival in cases of injury, and in population dynamic studies. The four parameter Weibull growth model is one of the nonlinear models is formed as ((Kamar, et al, (2020))

$$y_i = \beta_0 \, {x_i}^{\beta_1 - 1} e^{-\beta_2 {x_i}^{-\beta_3}}$$

Table (3) show the special cases of Weibull model.

Table (3): the special cases of Weibull model

case	the initial value of y_i
x_i=0	$y_i = 0$
$\beta_3 = 0$	$y_i = \beta_0 \, {x_i}^{\beta_1 - 1}$
$\beta_3 = 1$	$y_i = \beta_0 \, {x_i}^{\beta_1 - 1} e^{-\beta_2 x_i}$
$\beta_3 > 1$	has no inflexion point
$\beta_1 = 0$	$y_i = \frac{\beta_0}{x_i} \, e^{-\beta_2 {x_i}^{-\beta_3}}$
$\beta_1 = 1$	$y_i = \beta_0 \, e^{-\beta_2 {x_i}^{-\beta_3}}$
$\beta_2 = 0$	$y_i = \beta_0 \, {x_i}^{\beta_1 - 1}$
$\beta_2 = 1$	$y_i = \beta_0 \, {x_i}^{\beta_1 - 1} e^{-{x_i}^{-\beta_3}}$

3. Goodness of fit

For analyzing nonlinear models, there are several different methods (graphical and numerical) that show if the model has a good fit to the data or not. To identify which model was more efficient, Bias, mean squared error (MSE), root mean squared error (RMSE), mean absolute error (MAE), Bayesian information criterion (BIC), Coefficient of Determination R^2, Akaike's information criterion (AIC), Akaike's information criterion with sample size correction (AICc), and other numerical statistical indices were utilized. The form of each indication is shown in table (4).

Table (4): the form of each indication

Criteria	form
R^2	SSR/SST
F	$\frac{MSR}{MSE} = \frac{SSR/P}{SSE/(n-p-1)}$
RMSE	$RMSE = \sqrt{MSE}$
Bias	$Bias = \frac{1}{n}\sum_{i=1}^{n} e_i^2$
MAE	$MAE = \frac{1}{n}\sum_{i=1}^{n} \lvert e_i \rvert$
AIC	$AIC =$ n*ln (SSR/n)+2p
AIC_c	$AIC + \frac{2P(p+1)}{(n-p-1)}$
BIC	$BIC =$n*ln (SSR/n)+ n*ln (n)
MAPE	$\frac{1}{n}\sum_{r=1}^{n} \left\lvert \frac{\text{actual} - \text{Predict}}{\text{actual}} \right\rvert * 100$

were

$$SSR = \sum_{i=1}^{n}(y_i - \bar{y})^2 - \sum_{i=1}^{n}(y_i - \hat{y}_i)^2, SSE = \sum_{i=1}^{n} e_i^2 = \sum_{i=1}^{n}(y_i - \hat{y}_i)^2$$

A smaller value of AIC, AICc, BIC, RMSE, Bias and MAE

criteria indicate a preferable model. A bigger value of R^2,F also indicate a preferable model. To have a good forecasting, MAPE must be less than 20. (Moreno ,et al (2013))

Also, by evaluating the 95% $(1-\alpha)$ C.I., we can test the hypotheses about the models' parameters. the good fitted model has statistically significant parameter estimator at 5% level, when the C.I. of θ_j does not include zero.

4. Measures of nonlinearity

Because these models are nonlinear, nonlinearity metrics were utilized to assess the least squares estimators' statistical properties.

The curvature measures of Bates and Watts, the bias measure of Box, and the skewness measure of Hougaard are the most commonly used nonlinearity metrics. (Leite, (2018))

Curvatures of Bates and Watts

the parameter-effects nonlinearity Bates and Watts divide the concept of nonlinearity into two parts

a) Parameter-effects nonlinearity (PE)

On the leastsquares solution, (PE) is a measure of the lack of inequality and par allelism of parameter spacing lines.

b) Intrinsic nonlinearity (IN).

The curvature of the solution locus in sample space is measured by IN.IN is equal (0) in a linear regression model, because the solution locus is straight (a line, plane, or hyperplane).

The solution locus for a nonlinear regression model is **curved**, and IN measures the extent of that curvature.

(PE) and (IN) can be used to quantify the nonlinear regression model. As a result, if IN$< 1/\sqrt{F}$, within a 95 % confidence interval, the solution locus can be considered sufficiently linear.

if PE $< 1/\sqrt{F}$, The proposed parameter lines could be considered

sufficiently parallel and evenly spaced.

c) The bias measure of Box

The bias of estimates nonlinear regression model parameters can be determined using the Box bias metric. It revealed the disparity between the true values and parameter estimates.

A percentage bias =$\text{Bias}(\hat{\theta})\% = \frac{\text{Bias}(\hat{\theta})}{(\hat{\theta})} * 100$

Significant nonlinearity is defined as $\text{Bias}(\hat{\theta})\% > 1\%$ in absolute value.

d) The Hoggard measure of skewness

The skewness of estimates in a nonlinear regression model can be evaluated using Hoggard's skewness metric. Hougaard's measure of skewness can be used to examine the degree to which a parameter estimator exhibits nonlinear behavior as shown in table (5)

Table (5): Hoggard measure g_{1i}

g_{1i}	meaning
$g_{1i} < 0.1$	The estimator is very close to being linear.
$0.1 < g_{1i} < 0.25$	The skewness is easily noticeable.
$g_{1i} > 1$	indicates a considerable proportion of nonlinearity

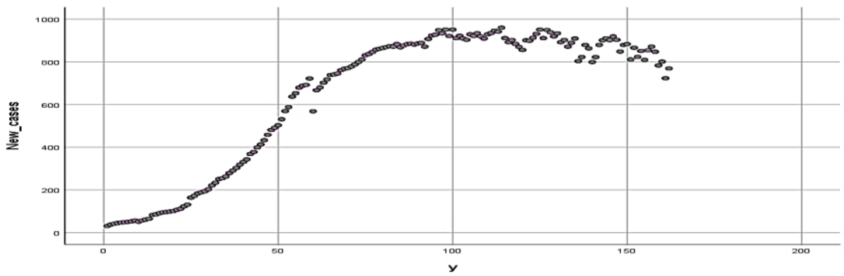
5. Application

This study depending on the daily number of COVID-19 cases in Egypt from 28th of July, 2021 to 5th January, 2022. from https:// WHO-COVID-19-global-data.csv

Models' estimation

The statistical package SPSS (26) was used to estimate the models for the daily COVID-19 new cases data and estimate the parameters. As shown in figure (1) the daily COVID-19 new cases data is looking to be non-linear

figure (1) the daily COVID-19 new cases data



as shown in Fig. (1), fourth wave Coronavirus affected by 4phases:

1. The first phase, start case infection and slow growth (31:279 cases) of the epidemic (from 28th July 2021to1st sept. 2021).
2. Second phase: fast growth infection (291:948). (From 2nd sept. 2021to31 th oct. 2021).
3. Third phase (from 1 st Nov. 2021to2 nd dec. 2021): steady-state and slow growth (peak) (933:938).
4. Fourth phase: (from 3 rd dec. 2021 to 4 th Jan.2022) start decrease (892:723).
5. (From 5 th Jan.2022 to 7 th Feb. 2022) The first phase, of Coronavirus affected fifth wave of (769:2301).

The daily COVID-19 new cases Descriptive Statistics are shown in table (6) .

Table (6) :The daily COVID-19 new cases Descriptive Statistics

Mean	Median	Variance	Std. Deviation
645.6	811.00	102390.8	319.98
Minimum	Maximum	Skewness	Kurtosis
31	960	-.845	-.931

The Maximum New cases =960 so, B_0 Starting value at all growth models will be 960 and all the starting values for studied growth models (daily COVID-19 new cases) are shown in table (7).

Table (7) : Starting values for studied growth models

parameter	β_0	β_1	β_2	β_3
Gompertz	960	-5	0.09	
Richards'	960	-0.05	2	3
Weibull	960	200	0.0009	3

Using the statistical package SPSS26 to estimate the models, table (8) showing the parameters predicted values for studied growth models (daily COVID-19 new cases) and its R Square, and table (9) showing the 95% Confidence Interval of parameters predicted values for the three models.

Table (8): parameters predicted values for studied growth models

Estimate	β_0	β_1	β_2	β_3	R^2
Linear	166.11	5.884			.742
Gompertz	903.885	-8.39	0.056		0.979
Richards	890.468	0.1	209.822	1.638	0.987
Weibull	890.112	837.084	0.000011	2.864	0.987

Table (9): the 95% Confidence Interval of parameters predicted values for studied growth models

Parameter Estimates(Weibull model)				
Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
β_0	890.112	4.315	881.588	898.635
β_1	837.083	10.327	816.686	857.481
β_2	1.112E-5	.000	1.488E-6	2.075E-5
β_3	2.864	.109	2.649	3.080

95% Confidence Interval(Gompertz model)

Parameter Estimates				
Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
β_0	903.886	6.176	891.688	916.083
β_1	-8.390	.722	-9.816	-6.964
β_2	.056	.002	.052	.061

Parameter Estimates(Richards' model)				
Parameter	Estimate	Std. Error	95% Confidence Interval	
			Lower Bound	Upper Bound
β_0	890.467	4.555	881.470	899.465
β_1	.100	.009	.082	.117
β_2	209.866	146.72	-79.914	499.645
β_3	1.638	.275	1.096	2.181

Except for Richards (β_2), the confidence interval of all model estimators does not include zero. This suggests that the Weibull model is the best models for representation of daily COVID-19 cases because all parameters' estimations of Weibull model are statistically significant at 5% level and it has the biggest R^2

From table (8), we can estimate the three models as shown in table (10).

Table (10): the estimate of the three models.

Gompertz	y=903.885*exp(-8.39*exp(-0.056*x))
Richards'	y=890.48/((1+209.822*exp(-0.1*x))$^{1/1.638}$
Weibull	y=890.112-837.08*exp(-0.000011*x))$^{2.864}$

After estimating the three models we must compute the Goodness of fit s indicators to identify the best model for daily COVID-19 new cases as shown in table (11)

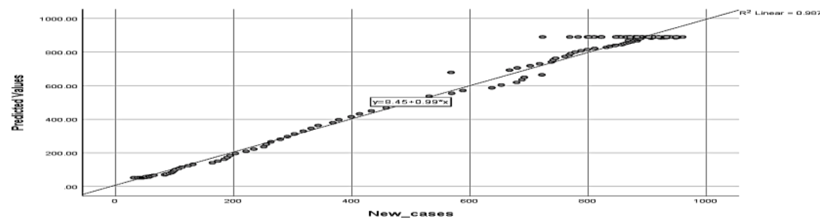
Table (11): Goodness of fit and criterion results of studied growth models.

Criteria	Gompertz	Richards	Weibull
R^2	0.979	0.987	0.987
RMSE	47.05333	37.35396	36.95301
Bias	2173.007	1360.862	1331.781
MAE	37.90809	25.17407	25.27778
AIC	1562.787	1486.972	737.315
AICc	1562.938	1487.225	737.5682
BIC	2068.978	1993.163	1243.506
MAPE	15.4722	.3822	5.3675

From Table (10), it was observed that:

1. The Weibull model gives lowest value of MAE=25.27 and RMSE =36.95301 which about 22% less than Gompertz model, and about 2% less than Richards' model, so the Weibull model provided the best fit
2. In other hand, Weibull model had lowest value of bias, this result reflects that the predicted of COVID-19 new-cases by Weibull model is very close (in mean) to actual new-cases as shown in Figure (2).

Figure (2): plot of actual cases of COVID-19 cases with predicated cases for Weibull model.

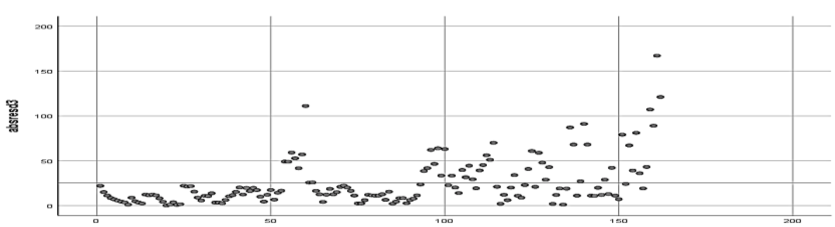


Normally and independent tests for Weibull model residuals.

Two basic assumptions about model residuals must be evaluated based on the examination of residuals using graphical approaches and statistical testing as follow:

Figure (3), shows the absolute values of Weibull model residuals displayed against time, we can observe that there is no pattern association between time and residuals, which means that the variance of residuals is homogeneous, or that the variance of the residuals is the same.

Figure (3): Homoscedasticity of Weibull model residual



1. L jung–Box Q test)

To test errors independence using the L jung–Box Q test

H0: Weibull model does not show a lack of fit

H1: Weibull model does show a lack of fit

The statistic Q, for a time series Y of length n can be formed as:

$$Q_k = n(n+2)\sum_{j=1}^{k}\frac{\hat{p}_j^2}{n-j}$$

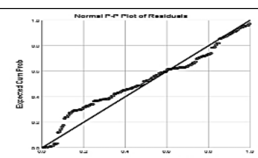
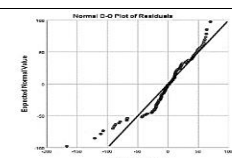
where $\hat{p}_j^2$ is the estimated autocorrelation of the series at lag j among fitted model residuals $e_1, e_2, \ldots, e_n$

For Weibull model, $Q_k = 617.939$ with sig (0), based on the asymptotic chi-square approximation, it means that Weibull model does show a lack of fit.

2. errors normally distribution with common variance

By plotting the model residuals against the normal CDF values, the (p–p) plot approach can be used to see if the errors follow a normal distribution. as seen in table (12) below.

Table (12); the **Weibull models'** (p–p) plot and Q-Q

Weibull model residuals p-p	Weibull model residuals Q-Q
	

the residuals are approximately normally distributed since the p–p normal plot and the Q-Q normal plot of residuals shows that the points lie near to the straight line.

6. Conclusions

In this study, the growth curve of real data sets (the daily number of fourth wave COVID-19 cases in Egypt) was examined using three sigmoidal growth models (Gompertz, Richards, and Weibull). Among the three models it was able to identify the Weibull Growth Model was the best growth model. It exhibited nearly linear behavior, and its estimated parameters are nearly unbiased, normally distributed, and have low variance. So, it recommends the Weibull Growth Model for further COVID-19 growth studies.

References

1. Al-Ani B., (2021). Statistical modeling of the novel COVID-19 epidemic in Iraq. Epidemiologic Methods, https://doi.org/10.1515/em-2020-0025.
2. Amar, L, A., Taha A. A., Mohamed. M.Y., (2020). Prediction of the final size for COVID-19 epidemic using machine learning: A case study of Egypt. Infectious disease modelling ,Vol (5), pp (622-634)
3. Archontoulis S. V., and Miguez F. E., (2015). Nonlinear Regression Models and Applications in Agricultural Research .Agronomy journal, Vol (107), p 786
4. Dagogo, J.; Nduka, E.C.; Ogoke, Uc. P., (2020). Comparative Analysis of Richards, Gompertz and Weibull Models. Journal of mathematics (IOSR-JM), Vol. 16, PP 15-25
5. Deif A. S., El-Naggar S. A., (2021). Modeling the COVID-19 spread, a case study of Egypt. Journal of the Egyptian mathematical society, vol (29), p (861).
6. Fahmy A. E.; Eldesouky M. M.; Mohamed A.S.A., , (2020). Epidemic Analysis of COVID-19 in Egypt, Qatar and Saudi Arabia using the Generalized SEIR. medRxiv preprint doi. https://doi.org/10.1101/2020.08.19.20178129.
   https://www.care.gov.eg/EgyptCare/index.aspx
   https://covid19.who.int/WHO-COVID-19-global-data.csv
7. Kamar S. H., and Msallam B. Sh., (2020). Comparative Study between Generalized Maximum Entropy and Bayes Methods to Estimate the Four Parameter Weibull Growth Model. journal of probability and statistics, Vol (2020), Article ID 7967345
8. Leite M. T., (2018). Sigmoidal Models Fitted to the Saccharomyces cerevisiae Growth Curve: Statistical Analysis Using Measures of Nonlinearity. American journal of engineering research, Vol (8), pp(1-9)
9. Mansour M. M., Farsi M. A., Mohamed S. M., Abd Elrazik E. M. (2021). Modeling the COVID-19 Pandemic Dynamics in Egypt and Saudi Arabia. Mathematics, Vol (9), p 827.
10. Moreno J. J. M., Pol A. P., Abad A. S., Blasco B. C.,(2013).Using the R-MAPE index as a resistant measure of forecast accuracy. Psicothema , Vol (25), No. 4, pp (500-506)

11. Radwan T., (2021), A statistical study of COVID-19 pandemic in Egypt. demonstration mathematica; vol (54), pp (233–244).
12. Rodr´ıguez G., (2010). Parametric Survival Models. https://data.princeton.edu/pop509/parametricsurvival.pdf
13. Shafii B., Price W. J., Swensen J. B., Murray G. A., (1991). nonlinear estimation of growth curve models for germination data analysis, https://newprairiepress.org/agstatconference/1991.
14. Sundar R. M., Palanivel M., (2017). Comparison of non-linear models to describe growth of cotton. international journal of statistics and applied mathematics; Vol (24), pp 86-93
15. Szabelska A., Siatkowski M., Goszczurna T., Zyprych-Walczak J., (2010). Comparison of growth models in package R. Nauka Przyroda Technologie, https://www.researchgate.net/publication/260036377.